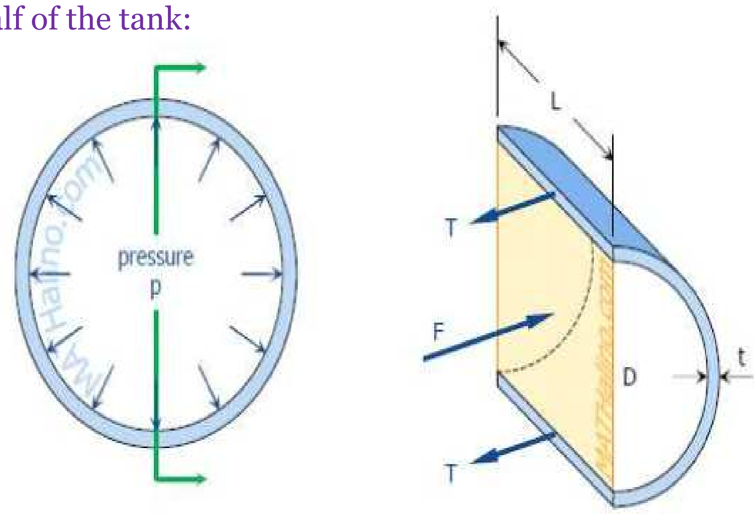
Thin-walled

A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

Tangential stresses

Consider the tank shown being subjected to an internal pressure p. The length of the tank is L and the wall thickness is t. Isolating the right half of the tank:



The forces acting are the total pressures caused by the internal pressure p and the total tension in the walls T.

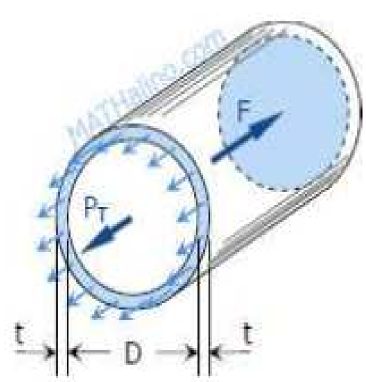
$$\left\{\begin{aligned} &F = pA = pDL \\ &T = \sigma_t A_{wall} = \sigma_t\, tL \\ &\Sigma F_H = 0 \\ &F = 2T \\ &pDL = 2(\sigma_t\, tL) \end{aligned}\right\} \quad \sigma_t = \frac{pD}{2t}$$

If there exist an external pressure po and an internal pressure pi, the formula may be expressed as:

$$\sigma_t = \frac{(p_i - p_o)D}{2t}$$

Longitudinal stresses

Consider the free body diagram in the transverse section of the tank:



The total force acting at the rear of the tank F must equal to the total longitudinal stress on the wall $P_T = \sigma_L A_{wall}$. Since t is so small compared to D, the area of the wall is close to πDt

$$\left\{\begin{aligned} F &= pA = p\frac{\pi}{4}D^2 \\ P_T &= \sigma_L \pi D t \\ \Sigma F_H &= 0 \\ P_T &= F \\ \sigma_L\, \pi D t &= p\frac{\pi}{4}D^2 \end{aligned}\right\}$$

$$\sigma_L = \frac{pD}{4t}$$

If there exist an external pressure p_o and an internal pressure p_i, the formula may be expressed as:

$$\sigma_L = \frac{(p_i - p_o)D}{4t}$$

It can be observed that the tangential stress is twice that of the longitudinal stress.

$$\sigma_t = 2\sigma_L$$

SPHERICAL SHELL

If a spherical tank of diameter D and thickness t contains gas under a pressure of $p = p_i - p_o$, the stress at the wall can be expressed as:

$$\sigma_t = \frac{(p_i - p_o)D}{4t}$$

Problem 1

A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m^2. (a) Calculate the tangential and longitudinal stresses in the steel. (b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m^2? (c) If the internal pressure were increased until the vessel burst, sketch the type of fracture that would occur.

Solution

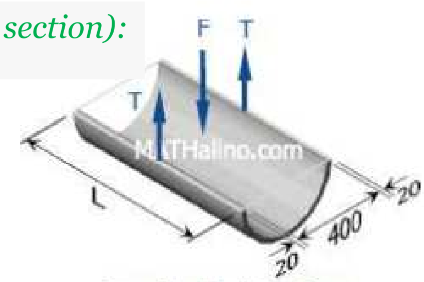
Part (a)Tangential stress (*longitudinal section):*

$$F = 2T$$

$$pDL = 2(\sigma_t t L)$$

$$\sigma_t = \frac{pD}{2t} = \frac{4.5(400)}{2(20)}$$

$$\sigma_t = 45\,\text{MPa} \qquad \textit{answer}$$



Longitudinal Section

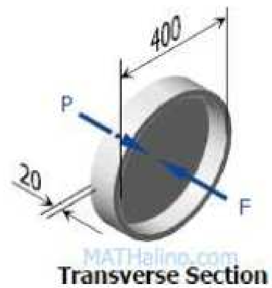
Longitudinal Stress (*transverse section*):

$$F = P$$

$$\tfrac{1}{4}\pi D^2 p = \sigma_l(\pi D t)$$

$$\sigma_l = \frac{pD}{4t} = \frac{4.5(400)}{4(20)}$$

$$\sigma_l = 22.5\,\text{MPa} \qquad answer$$



Part (b)

From (a), $\sigma_t = pD/2t$ and $\sigma_l = pD/4t$ thus, $\sigma_t = 2\sigma_l$, this shows that tangential stress is the critical.

$$\sigma_t = \frac{pD}{2t}$$

$$120 = \frac{p(400)}{2(20)}$$

$$p = 12\,\text{MPa} \qquad answer$$

The bursting force will cause a stress in the longitudinal section that is twice to that of the transverse section. Thus, fracture is expected as shown.